Arithmetic Sequences +Series

- An alternative name for an arithmetic sequence is an arithmetic progression.
- This is often abbreviated to A.P.
- 1^{st} term = $a_1=a$
- no. of terms=n
- Last term=$a_n=l$
- CD=d
- $a_k=k^{th}$ term

- How many numbers are in the following sequence?
- 7,9,11,13,…,375

l=a+(n-1)d

375=7+(n-1)2

375=7+2n-2

375-5=2n

n=175

Sum of terms in an AP

- $S = \frac{1}{2} n[2a+(n-1)d]$
- $S = \frac{1}{2} n(a+l)$ since $l=a+(n-1)d$
- a=1st term
- d=common difference
- n=number of terms

Why $S = \frac{1}{2} n[2a+(n-1)d]$?

$S=a+(a+d)+\ldots+(a-(n-2)d)+(a+(n-1)d)$

write it backwards

$S=(a+(n-1)d)+(a+(n-2)d)+\ldots+(a+d)+a$

$2S=(2a+(n-1)d)+(2a+(n-1)d)+\ldots.$

There are n of these terms.

$2S=n(2a+(n-1)d)$

$S= \frac{1}{2} n(2a+(n-1)d)$

Examples

- Find the 20th term in the sequence 12,8,4,….

  $a=12 \quad d=-4$

  $a_k=a+(n-1)d$

  $=12+(20-1)x-4$

  $=12+19x-4$

  $=12-76$

  $=-64$

- Find the value of $21+17+13+\ldots+(-23)$

  $a=21$ $d=-4$ $l=-23$ $n=?$

  $-23=21+(n-1)x-4$

  rearrange/solve to find $n=12$

$S= \frac{1}{2} n(2a+(n-1)d)$

$S= \frac{1}{2} x\ 12(2x21+(12-1)x-4)$

$S=6(42+-44)$

$S=-12$

- A teaching job has a starting salary of £18000 per year with an annual increase of £1000. Find:
- Their salary in the 15th year.
- The length of time she has to work to earn £1000000

- $a=18000$ $\quad d=1000$

- $a_{15} = a + (n-1)d$

  $= 18000 + (15-1) \times 1000$

  $= 18000 + 14000$

  $= £32000$

- $S = \frac{1}{2} n(2a+(n-1)d)$
- $a=18000$ $d=1000$ $s=1000000$

- $1000000 = \frac{1}{2} x n(2x18000+(n-1)x1000)$

$$= \frac{1}{2} n(36000+1000n-1000)$$

$500n^2+17500n-1000000=0$

$n^2+35n-2000=0$

Solve using the quadratic formula to get

x=30.5 or -65.5. So x=30.5